Resurgent ZZ and FZZT branes in minimal strings and JT gravity

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Physical resurgence: On quantum, gauge, and stringy

Based on:

- [PG, Ricardo Schiappa] arXiv:2108.11409
- [B. Eynard, E. Garcia-Failde, PG, D. Lewański, Ricardo Schiappa] arXiv:2210.xxxxx

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Motivation

- Can we classify and systematically compute non-perturbative data in 2-d gravitational theories?
- Can we use such data to reproduce known behaviours of observables (e.g. spectral density, spectral form factor)?
- A large class of models (minimal strings, JT gravity) admits a matrix model description: combine topological recursion and resurgence!

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Jackiw-Teitelboim Gravity

- 2d dilaton gravity with action [Jackiw-Teitelboim]

$$S_{\rm JT} = -\frac{S_0}{4\pi}\underbrace{\int_{\mathcal{M}}\sqrt{g}R}_{\text{topological}} - \frac{1}{2}\underbrace{\int_{\mathcal{M}}\sqrt{g}\phi(R+2)}_{\text{dilaton action}} + (\text{boundary terms})$$

- Dilaton ϕ acts as Lagrange multiplier: $R=-2 \;\rightarrow\; \mathrm{AdS}_2$
- Different topologies weighted by $\left(\mathrm{e}^{S_0}\right)^{2-2g-n} = g_{\mathrm{s}}^{2g+n-2}$
- Holographic dual of SYK model → random ensemble of quantum mechanical models → random matrices

[Sachdev-Ye, Kitaev]
[Saad-Shenker-Stanford]

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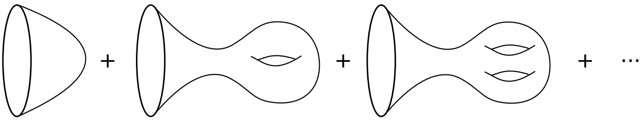
Euclidean Partition Functions

- Relevant quantities for holography: Euclidean partition functions

$$\langle Z(\beta_1)\cdots Z(\beta_n)\rangle \simeq \sum_{g=0}^{\infty} g_{\rm s}^{2g+n-2} Z_{g,n}(\beta_1)\cdots Z(\beta_n)$$

- Surfaces with n Schwarzian boundaries + g handles



- Weil–Petersson volumes are the building blocks of EPFs:

[Saad-Shenker-Stanford]

$$\langle Z(\beta)\rangle \simeq g_s^{-1} Z_{\rm disk}(\beta) + \sum_{g=1}^{\infty} g_s^{2g-1}\int_0^{\infty} bdb\, V_{g,1}(b) Z_{\rm trumpet}(\beta,\, b)$$

- $V_{g,n} \sim (2g)!$ [Mirzakhani-Zograf] $\to$ Resurgence!

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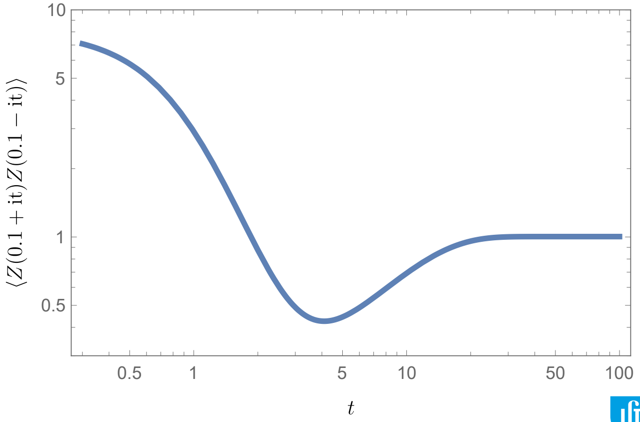
The Spectral Form Factor

- Two-boundary EPF of particular interest: spectral form factor:
  $\langle Z(\beta + \mathrm{i}t) Z(\beta - \mathrm{i}t) \rangle$ connected + disconnected
- Airy example:



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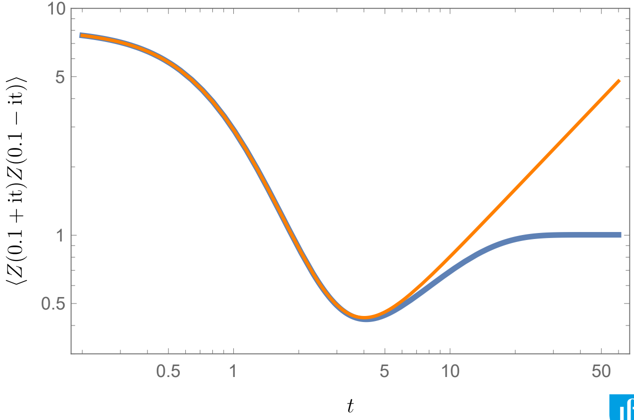

- Nonperturbative effects are needed!

Outline

1. Matrix Models
2. Non-perturbative effects in matrix models
3. Resurgence toolkit
4. Borel plane singularities
5. Large order checks
6. Resummations
7. Summary and outlook

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Table of Contents

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Review of matrix models

- $N \times N$ Hermitian one-matrix model with potential $V(x)$

$$Z_N = \frac{1}{\mathrm{vol}\,(\mathrm{U}(N))} \int \mathrm{d}M \mathrm{e}^{-\frac{1}{g_{\mathrm{s}}} \mathrm{Tr} V(M)}$$

- Associated spectral curve (one-cut case):

$$y(x) = M(x) \sqrt{(x-a)(x-b)}$$

related to the holomorphic effective potential acting on eigenvalues

$$V'_{\mathrm{h;eff}}(x) = y(x)$$

and to the spectral density of eigenvalues

$$\rho_0(\lambda) = \frac{1}{2\pi} \operatorname{Im} y(\lambda)$$

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Correlators and topological recursion

- Matrix model correlators:

$$W_n(z_1,\dots,z_n) = 2^n z_1\cdots z_n \left\langle \mathrm{Tr}\,\frac{1}{z_1^2-M}\cdots \mathrm{Tr}\,\frac{1}{z_n^2-M}\right\rangle_{\text{(conn)}}$$

have a perturbative expansion

$$W_n(z_1,\dots,z_n) \simeq \sum_{g=0}^{+\infty} W_{g,n}(z_1,\dots,z_n)\, g_{\mathsf{s}}^{2g+n-2}$$

- They are computed by topological recursion [Eynard-Orantin]:

$$W_{g,n}(z_1,J) = \operatorname*{Res}_{z\to\alpha}\Bigg\{K_y(z_1,z)\Big[W_{g-1,h+1}(z,-z,J) + \\ + \sum_{\substack{m+m'=g\\ I\sqcup I'=J}}{}^{\prime}\; W_{m,|I|+1}(z,I)\,W_{m',|I'|+1}(-z,I')\Big]\Bigg\}$$

The dual matrix model of JT gravity

- From disk amplitude of the JT path integral: spectral density of dual matrix model:

[Stanford-Witten]
[Saad-Shenker-Stanford]

$$\rho_0(E) = \frac{1}{4\pi^2}\sinh 2\pi\sqrt{E}$$

- From this, Mirzakhani spectral curve: $\frac{\sin 2\pi\sqrt{x}}{4\pi}$
- Infinite cut $\to$ double scaled matrix model
- The $W_{g,n}(z_1, \ldots, z_n)$, $V_{g,n}(b_1, \ldots, b_n)$, and $Z_{g,n}(\beta_1, \ldots, \beta_n)$ are related by a web of integral transforms

[Saad-Shenker-Stanford]
[Eynard-Orantin]

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$(2, 2k+1)$ minimal strings

- The JT gravity spectral curve can be seen as the $k \to \infty$ limit of a class of spectral curves:

$$y_{(2,2k-1)}(x) = T_{2k-1}(\sqrt{x}) \tag{1}$$

- These are the spectral curves of $(2, 2k-1)$ minimal strings: Liouville gravity coupled to a $(2, 2k-1)$ minimal model CFT
- First examples:

$$\begin{aligned} y_{(2,1)}(x) &= \sqrt{x} && \text{Airy curve} \\ y_{(2,3)}(x) &= T_3(\sqrt{x}) && \text{Painlevé curve} \\ &\dots && \dots \end{aligned}$$

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Table of Contents

1. Matrix Models
2. Non-perturbative effects in matrix models
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5. Large order checks
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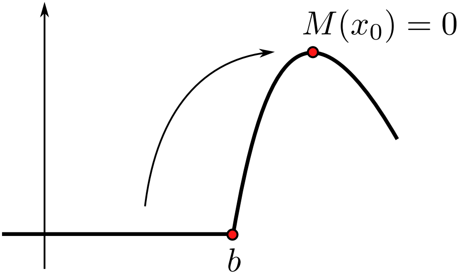
ZZ branes in matrix models



- ZZ branes are associated to eigenvalue tunneling
- One-instanton contribution of the form [David, Mariño-Schiappa-Weiss]

$$F^{(1)} \simeq g_s^{1/2} S_1 \exp\left(-\frac{A}{g_s}\right) \sum_{n=0}^{\infty} F_{n+1}^{(1)} g_s^n$$

- All non-perturbative data captured by spectral geometry

ZZ branes from the spectral geometry

- For example, the instanton action:

$$A = V_{\mathrm{h;eff}}(x_0) - V_{\mathrm{h;eff}}(b) = \int_b^{x_0} y(x)\mathrm{d}x$$

- In the case of $(2, 2k-1)$ minimal strings: $(k-1)$ non-trivial saddles [Seiberg-Shih]

$$A_{(n,k)} = (-1)^{k+n} \left(\frac{1}{2k+1} + \frac{1}{2k-3} \right) \sin \frac{2\pi n}{2k-1}$$

- But, resonance! Instanton actions are actually twice as many [PG-Schiappa]

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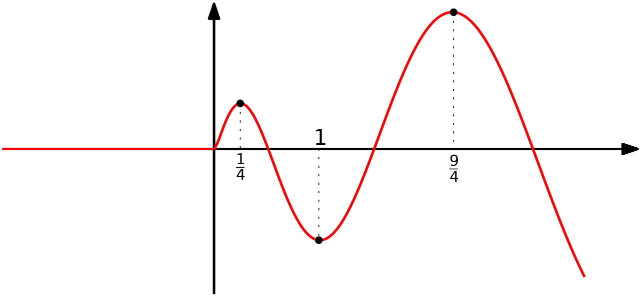
ZZ branes from the spectral geometry

- In the case of JT gravity:



- Infinitely many saddles!

$$A_\ell = \int_0^{\ell^2/4} y(x)\mathrm{d}x = \frac{(-1)^{\ell+1}}{4\pi^2}$$

- The remaining data is captured by a saddle-point integral passing through the non-trivial saddle x_0

$$F^{(1)} = \frac{1}{2\pi} \int_{\Gamma_0} \psi(x) \mathrm{d}x$$

where $\psi(x)$ is constructed via topological recursion

[Eynard-GarciaFailde-PG-Lewanski-Schiappa]

$$\psi(z^2) \equiv \exp\left(\sum_{g=0,n=1}^{\infty} \frac{g_s^{2g+n-2}}{n!} \overbrace{\int_{-z}^{z} \cdots \int_{-z}^{z}}^{n} W_{g,n} \right)$$

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ZZ brane data

- For example, one-loop around one-instanton for the $(2, 2k-1)$ minimal string: [PG-Schiappa]

$$S_1 \cdot F_1^{(1)} = \frac{1}{4\sin\frac{n\pi}{2k-1}} \sqrt{\frac{(-1)^{k+n+1}}{2\pi\,(2k-1)}\cot\frac{n\pi}{2k-1}}.$$

- The saddle point integral can be computed algorithmically
- We easily get many loops around the one-instanton configuration (JT) [Eynard-GarciaFailde-PG-Lewanski-Schiappa]:

$$S_1 \cdot F_1^{(1)} = \frac{\mathrm{i}}{\sqrt{2\pi}}, \qquad \widetilde{F}_2^{(1)} = -\frac{68}{3} - \frac{5\pi^2}{6},$$

$$\widetilde{F}_3^{(1)} = \frac{12104}{9} + \frac{818\pi^2}{9} + \frac{241\pi^4}{72},$$

$$\widetilde{F}_4^{(1)} = -\frac{10171120}{81} - \frac{311672\pi^2}{27} - \frac{175879\pi^4}{270} - \frac{163513\pi^6}{6480} - \frac{29\pi^8}{48},$$

$$\widetilde{F}_5^{(1)} = \frac{3859832480}{243} + \frac{442580824\pi^2}{243} + \frac{50891471\pi^4}{405} + \frac{33364187\pi^6}{4860} + \frac{9595009\pi^8}{31104} + \frac{19613\pi^{10}}{1440}$$

$\ldots$

- Up to 12 loops!

ZZ branes for correlators

- The non-perturbative topological recursion construction generalizes to correlators via the loop insertion operator

$$\Delta_x W_n(x_1, \ldots, x_n) = g_s W_{n+1}(x, x_1, \ldots, x_n)$$

- Completely new non-perturbative data for correlators, which was out of reach with other approaches (e.g. string equations)
- For example [Eynard-GarciaFailde-PG-Lewanski-Schiappa]:

$$S_1 \cdot \widetilde{W}^{(1)}_{1,n}(z_1, \ldots, z_n) = \prod_{i=1}^{n} \frac{4}{4z_1^2 - 1},$$

- Universal!

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- FZZT branes are associated to determinant insertions (i.e. orthogonal polynomials):

$$\Psi(x) = \mathrm{e}^{-\frac{1}{2g_s}V(x)} \langle \det(x - M) \rangle = \exp\left(-\frac{V_{\mathrm{h;eff}}(x)}{2g_s}\right) \sum_{n=0}^{\infty} \Phi_{n+1}(x) g_s^n$$

- The instanton action is the holomorphic effective potential → x-dependent instanton action!
- They contribute only to correlators: not seen in the free energy
- Also computable via topological recursion [Eynard-Orantin]:

$$\Psi(z^2) = \exp\left(\sum_{g=0,n=1}^{\infty} \frac{g_s^{2g+n-2}}{n!} \overbrace{\int_{\infty}^{z} \cdots \int_{\infty}^{z}}^{n} W_{g,n} \right)$$

Table of Contents

1 Matrix Models

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3 Resurgence toolkit

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Transserier

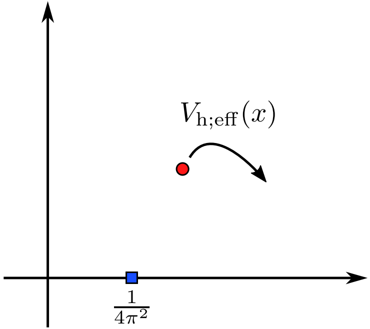
- The generic observable $\mathcal{O}$ is described by a transseries, which will contain both ZZ and FZZT contributions:

$$\mathcal{O} = \mathcal{O}^{(0)} + \mathcal{O}^{(ZZ)} + \mathcal{O}^{(FZZT)}$$

- Not the full story! Missing multi-instanton contributions + resonance (which is general in these models)
- Enough to capture expected non-perturbative effects and leading large genus asymptotics

Borel plane singularities

- We expect two ZZ brane singularities at $\pm\frac{1}{4\pi^2}$, and two FZZT brane singularities at $\pm V_{\mathrm{h;eff}(x)}$



- The FZZT singularity moves around as x changes!

Resurgent large order relations

- Instanton sectors attached to singularities in the Borel plane
- Cauchy's theorem gives us large order relation (example for two singularities):

$$\mathcal{O}_g^{(0)} \simeq \frac{S_1 \mathcal{O}_1^{(1)}}{2\pi \mathrm{i}} \frac{\Gamma(2g-\beta_1)}{A_1^{2g-\beta_1}} \left(1 + \frac{A_1}{2g-\beta-1} \frac{\mathcal{O}_2^{(1)}}{\mathcal{O}_1^{(1)}} + O(g^{-2})\right) + \\ + \frac{S_2 \mathcal{O}_1^{(2)}}{2\pi \mathrm{i}} \frac{\Gamma(2g-\beta_2)}{(A_2)^{2g-\beta_2}} \left(1 + \frac{A_2}{2g-\beta_2-1} \frac{\mathcal{O}_2^{(2)}}{\mathcal{O}_1^{(2)}} + O(g^{-2})\right) + \cdots$$

- Singularity that is closest to the origin dominates the asymptotics
- Large g asymptotics entirely encoded in non-perturbative data

1. Numerical checks of our computations
2. Large g asymptotics of quantities of interest

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Table of Contents

1. Matrix Models
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3. Resurgence toolkit
4. Borel plane singularities
5. Large order checks
6. Resummations
7. Summary and outlook

The one point function: integral representation

Since the FZZT instanton actions depend on the correlators variables z_i, we expect them to move around in the complex plane as the z_i change.

The position of Borel-plane singularities can be captured through approximate Borel transforms. One way of obtaining them is by making use of the integral representations of the correlators. For the one-point function we have:

$$\begin{aligned} W_1^{(1)}(g_s; z) &\simeq -\frac{1}{2\mathrm{i}} \int_{\mathcal{I}} \mathrm{d}x \frac{1}{x - z^2} \frac{1}{\sqrt{x}} \exp\left(-\frac{V_{\text{eff}}(x)}{g_s}\right) \\ &\simeq -\frac{1}{2\mathrm{i}} \int_{\widetilde{\mathcal{I}}} \mathrm{d}s \frac{1}{V'_{\text{eff}}\left(x\left(s\right)\right)} \frac{1}{x\left(s\right) - z^2} \frac{1}{\sqrt{x\left(s\right)}} \exp\left(-\frac{s}{g_s}\right) \end{aligned}$$

The integrand is intepreted as an ‘approximate Borel transform’, featuring both the ZZ and FZZT Borel-plane singularities.

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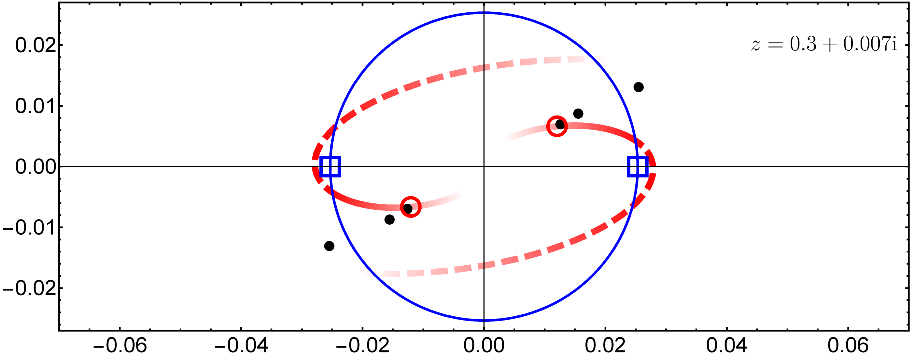
Otherwise, we can make use of Padé approximants:



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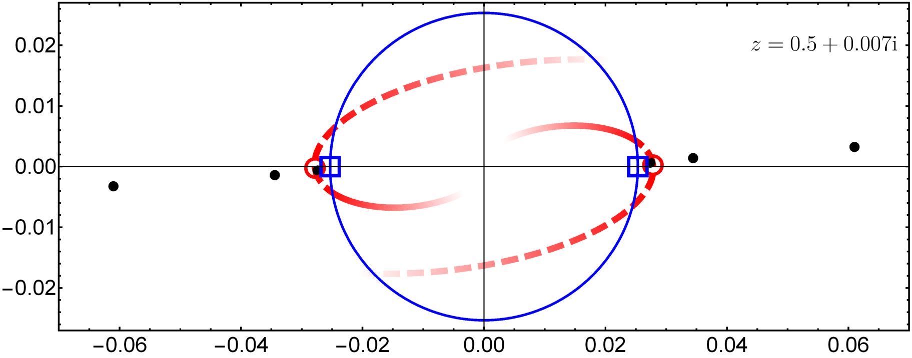

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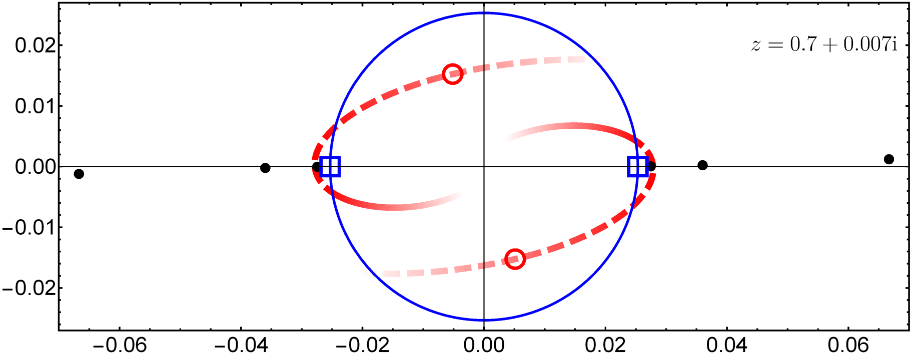

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The two point function: integral representation

The story is very similar for the two-point function, but here from the integral representation we get two distinct FZZT brane singularities:

$$W_2^{(1)}\left(g_s; z_1, z_2\right) \simeq \frac{1}{\mathrm{i}} \int_{\widetilde{\mathcal{I}}} \mathrm{d}s \frac{1}{V_{\text{eff}}'\left(x\left(s\right)\right)} \frac{1}{x\left(s\right) - z_1^2} \frac{1}{x\left(s\right) - z_2^2} \exp\left(-\frac{s}{g_s}\right)$$

depending on the two variables z_1 and z_2.

This is corroborated by the Padé approximant analysis.

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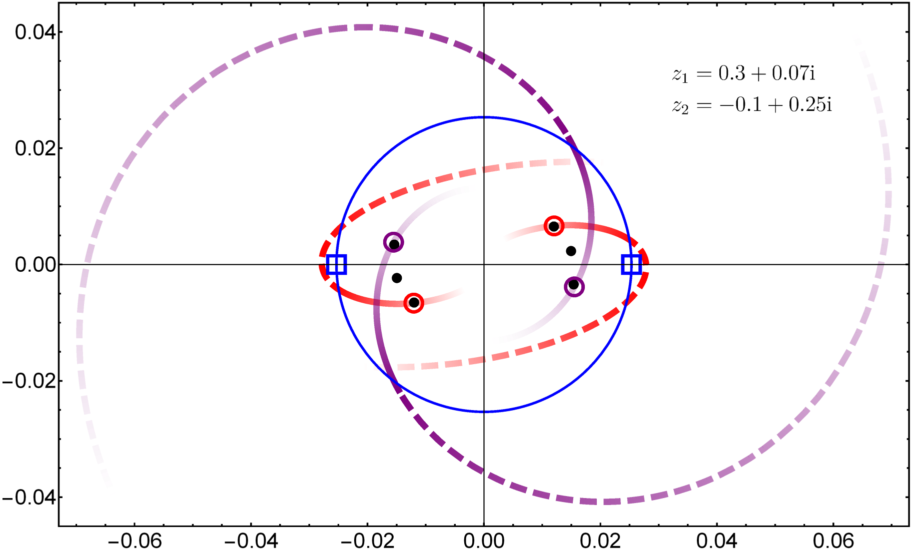
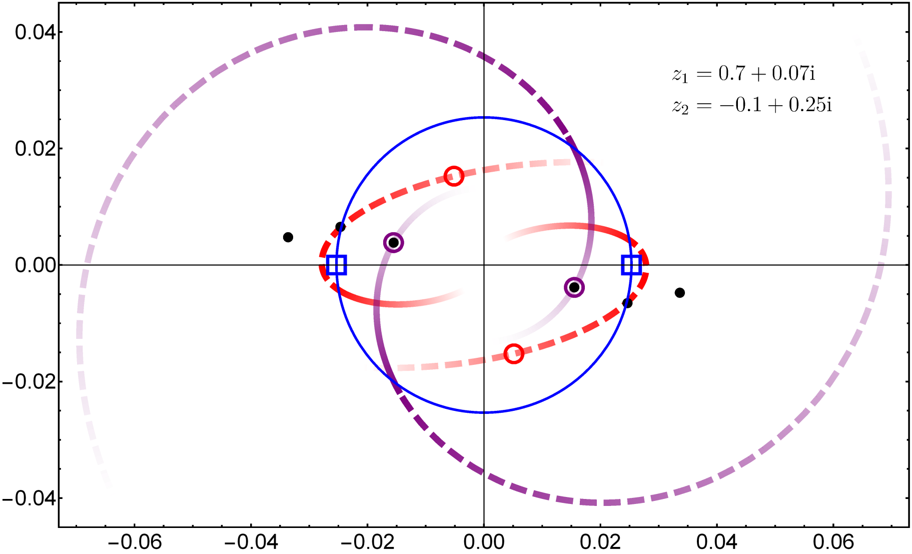
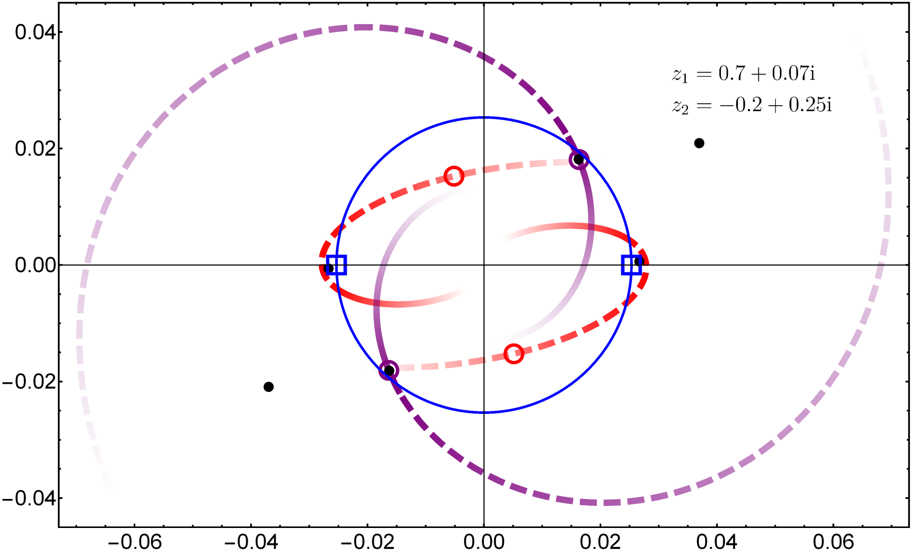
The two point function: Padé approximants



The two point function: Padé approximants



The two point function: Padé approximants



The two point function: Padé approximants

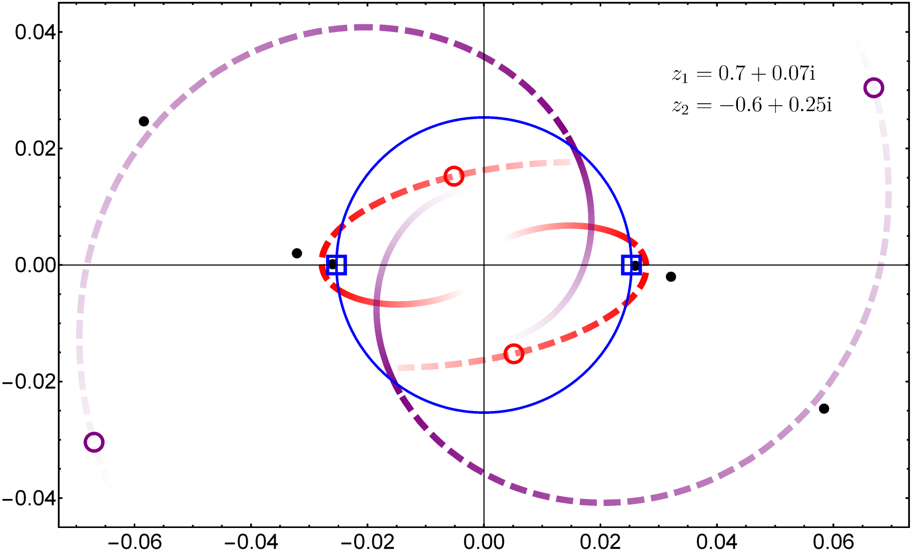


Table of Contents

1. Matrix Models
2. Non-perturbative effects in matrix models
3. Resurgence toolkit
4. Borel plane singularities
5. Large order checks
6. Resummations
7. Summary and outlook

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- Only ZZ brane non-perturbative effects
- We can generate many Weil–Petersson volumes with Zograf's algorithm
- From them, we construct sequences which at $g \to \infty$ converge to the non-perturbative coefficient we want to test:

$$\frac{V_{g+1,0}}{4g^2 V_{g,0}} = \frac{1}{A^2}\left(1 + \frac{1-2\beta}{2g} + O(g^{-2})\right)$$

$$2g\left(A^2 \frac{V_{g+1,0}}{4g^2 V_{g,0}} - 1\right) = 1 - 2\beta + O(g^{-1})$$

$$\frac{A^{2g-\beta} V_{g,0}}{\Gamma(2g-\beta)} = \frac{S_1 F_1^{(1)}}{2\pi i}\left(1 + O(g^{-1})\right)$$

and so on.

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Free energy: the instanton action

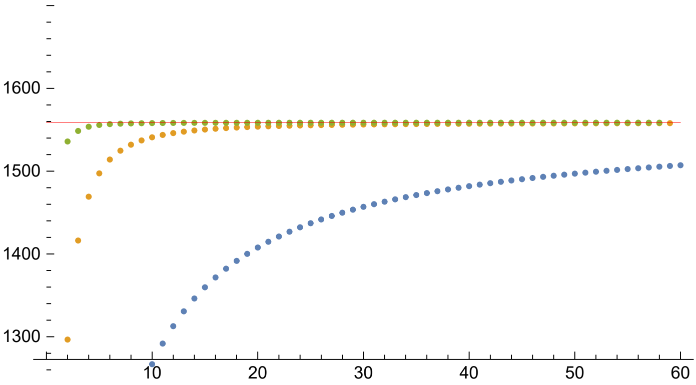

Figure: The sequence $\frac{V_{g+1,\,0}}{4g^2 V_{g,\,0}}$ (blue), its first two Richardson transforms (orange and green),and the predicted value $1/A^2 = 16\pi^2$ (red).

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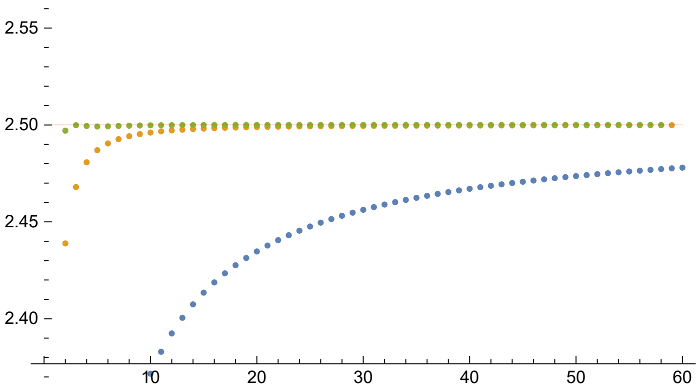

Figure: The sequence $2g\left(A^2 \frac{V_{g+1,\,0}}{4g^2 V_{g,\,0}} - 1\right)$ (blue), its first two Richardson transforms (orange and green), and the predicted value $\beta = 5/2$ (red).

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Free energy: the one-loop around one-instanton

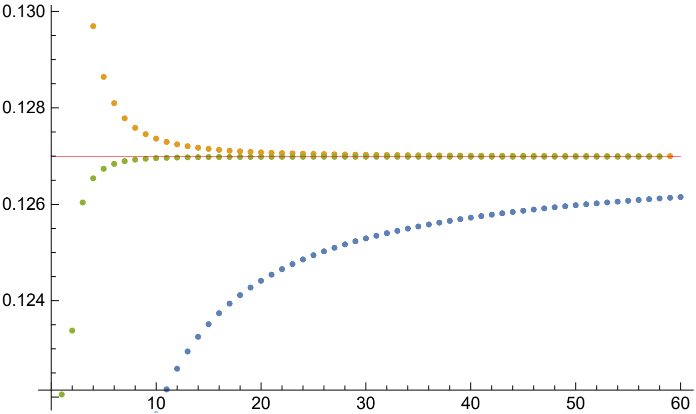

Figure: The sequence $\frac{A^{2g-\beta}F_g^{(0)}}{\Gamma(2g-\beta)}$ (blue), its first two Richardson transforms (orange and green) and the predicted value $\frac{S_1 F_1^{(1)}}{2\pi \mathrm{i}} = \frac{1}{\sqrt{2}\pi^{3/2}}$ (red).

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The one point function

- Both FZZT and ZZ brane contributions:

$$W_1(z) = W_1^{(0)}(z) + W_1^{(ZZ)}(z) + W_1^{(FZZT)}(z),$$

- Competing large genus asymptotics

$$W_{g,1}^{(0)}(z) \simeq \frac{1}{\sqrt{2}\pi^{\frac{3}{2}}}(4\pi^2)^{2g-\frac{3}{2}}\Gamma\left(2g-\frac{3}{2}\right)\left[\frac{4}{z^2-4}+\dots\right]+\dots$$
$$+\frac{1}{\pi}\left(V_{\text{eff}}\left(z^2\right)\right)^{-2g+1}\Gamma\left(2g-1\right)\left[\frac{1}{2z}+\dots\right]+\dots$$

- Perturbative data generated through TR (slow)

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One-point function: the instanton action

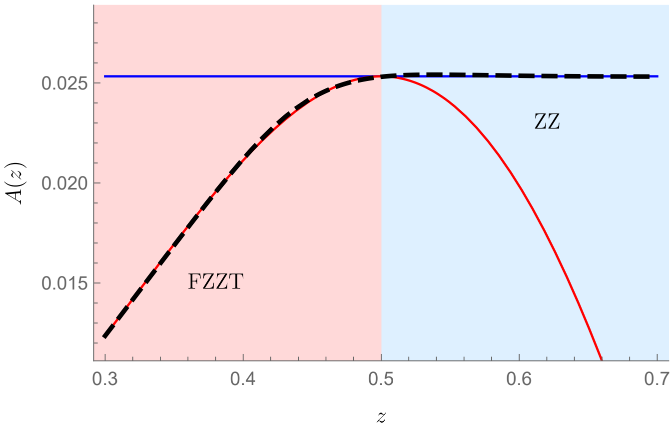

Figure: The black dashed line is the fifth Richardson tranform of the instanton action sequence of $W_1(z)$, as a function of z. In red and blue, the theoretical values associated to FZZT and ZZ branes.

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One-point function: the characteristic exponent

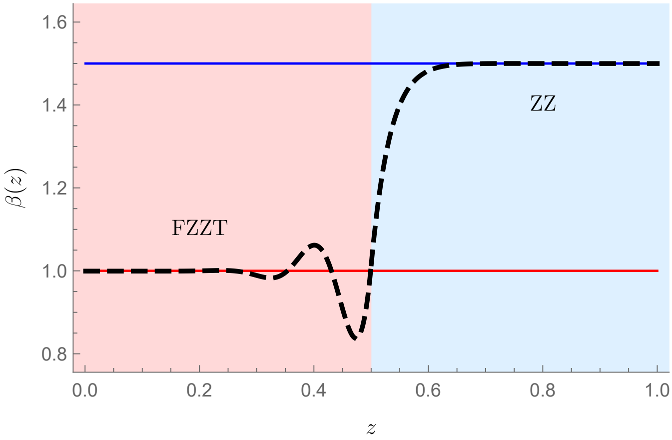

Figure: The black dashed line is the fifth Richardson tranform of the characteristic exponent sequence of $W_1(z)$, as a function of z. In red and blue, the theoretical values associated to FZZT and ZZ branes.

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One-point function: the one-loop around one-instanton

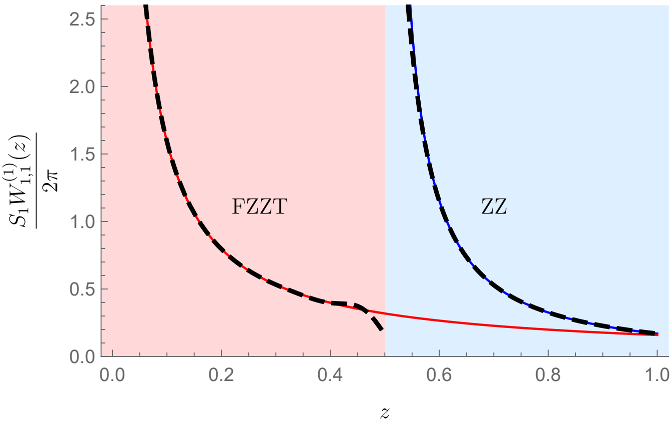

Figure: The black dashed line is the fifth Richardson tranform of the one-loop around one-instanton sequence of $W_1(z)$, as a function of z. In red and blue, the theoretical values associated to FZZT and ZZ branes.

The two-point function

- Two distinct FZZT brane contributions:

$$W_2(z_1,z_2) = W_2^{(0)}(z_1,z_2) + W_2^{(ZZ)}(z_1,z_2) + W_2^{(FZZT_1)}(z_1,z_2) + \\ + W_2^{(FZZT_2)}(z_1,z_2),$$

- Competing large genus asymptotics:

$$W_{g,2}^{(0)}(z_1,z_2) \simeq \frac{1}{\sqrt{2}\pi^{\frac{3}{2}}}(4\pi^2)^{2g-\frac{1}{2}}\Gamma\left(2g-\frac{1}{2}\right)\left[\frac{4}{z_1^2-4}\frac{4}{z_2^2-4}+\dots\right]+\dots \\ +\frac{1}{\pi}\left(V_{\text{eff}}\left(z_1^2\right)\right)^{-2g}\Gamma(2g)\left[\frac{1}{z_2^2-z_1^2}+\dots\right]+\dots \\ +\frac{1}{\pi}\left(V_{\text{eff}}\left(z_2^2\right)\right)^{-2g}\Gamma(2g)\left[\frac{1}{z_1^2-z_2^2}+\dots\right]+\dots$$

- Perturbative data generated through TR (slow)

Two-point function: the instanton action

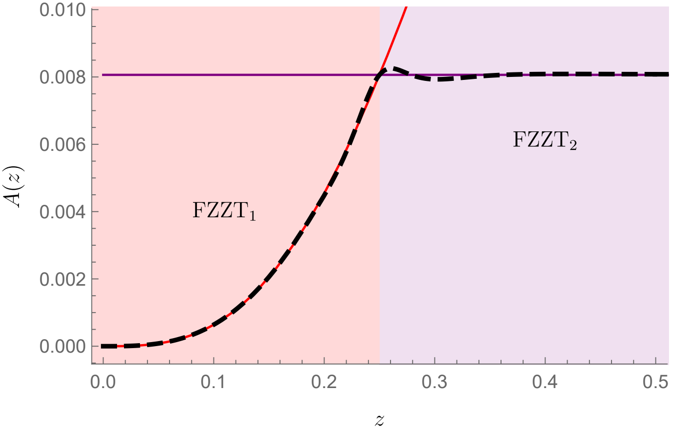

Figure: The fourth Richardson tranform of the instanton action sequence for $W_2(z, 0.25)$, as a function of z. In red and violet, the theoretical values associated to the FZZT_1 and FZZT_2 instanton actions.

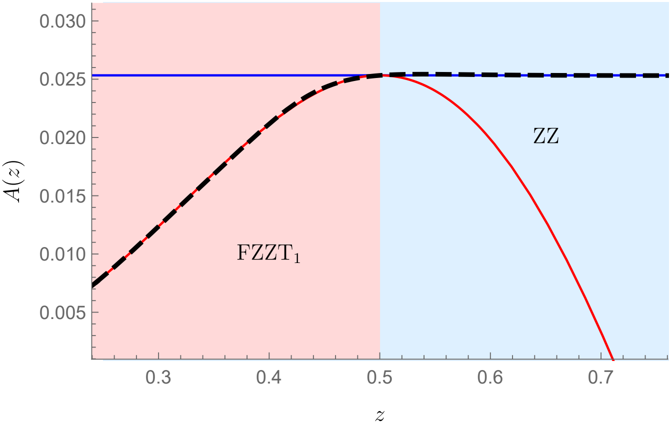

Figure: The fourth Richardson tranform of the instanton action sequence for $W_2(z, 0.75)$, as a function of z. In red and blue, the theoretical values associated to the FZZT_1 and ZZ instanton actions.

Two-point function: the characteristic exponent

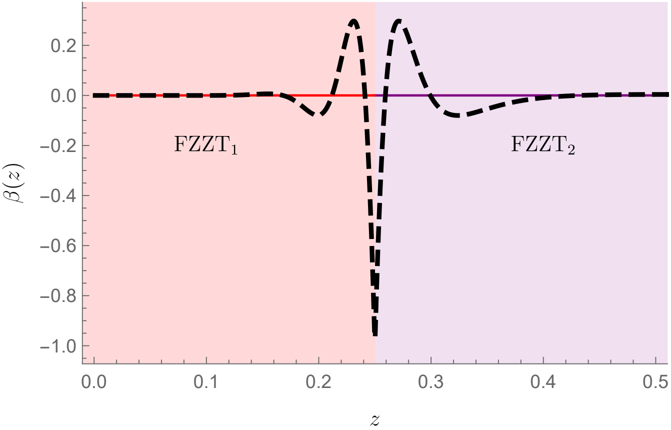

Figure: The fourth Richardson tranform of the characteristic exponent sequence for $W_2(z, 0.25)$, as a function of z. In red and violet, the theoretical values associated to the FZZT$_1$ and FZZT$_2$ characteristic exponents.

Two-point function: the characteristic exponent

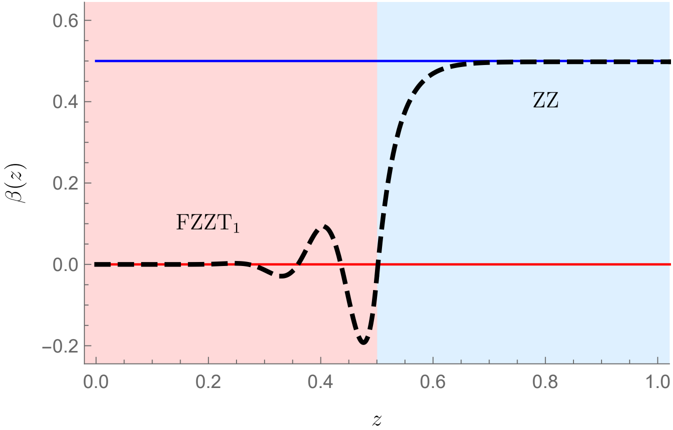

Figure: The fourth Richardson tranform of the characteristic exponent sequence for $W_2(z, 0.75)$, as a function of z. In red and blue, the theoretical values associated to the FZZT_1 and ZZ characteristic exponents.

Two-point function: the one-loop around one-instanton

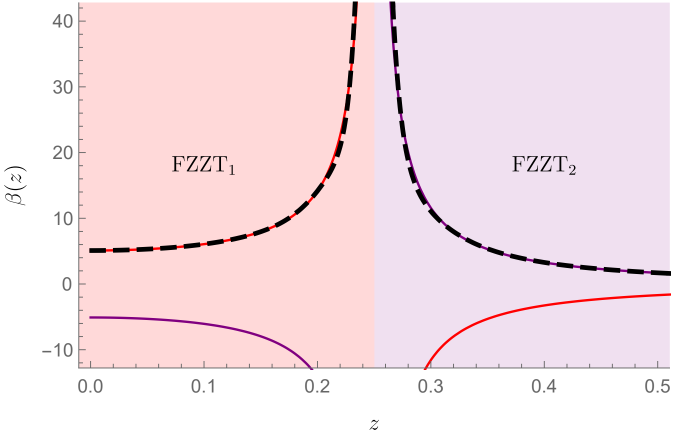

Figure: The fourth Richardson tranform of the one-loop around one-instanton sequence for $W_2(z, 0.25)$, as a function of z. In red and violet, the theoretical values associated to the FZZT$_1$ and FZZT$_2$ one-loop around one-instanton.

Two-point function: the one-loop around one-instanton

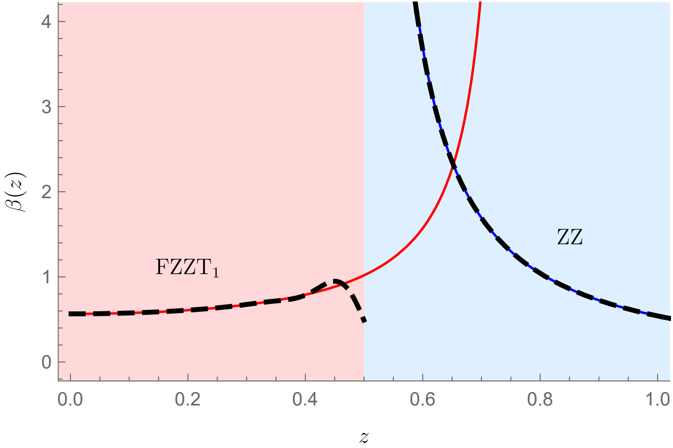

Figure: The fourth Richardson tranform of the one-loop around one-instanton sequence for $W_2(z, 0.75)$, as a function of z. In red and blue, the theoretical values associated to the FZZT$_1$ and ZZ one-loop around one-instanton.

Table of Contents

1 Matrix Models

2 Non-perturbative effects in matrix models

3 Resurgence toolkit

4 Borel plane singularities

5 Large order checks

6 Resummations

7 Summary and outlook

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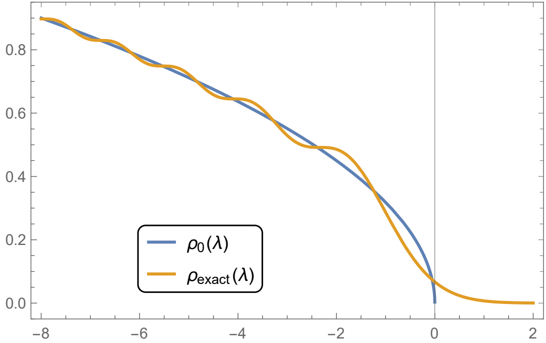
The spectral density

- In the Airy toy model ($y(x) = \sqrt{x} \implies \rho_0(\lambda) = \sqrt{-x}/(2\pi)$), we have an exact formula for the resummed spectral density:

$$\rho(\lambda) = g_s^{-2/3} \left[\mathrm{Ai}' \left(\lambda g_s^{-2/3} \right)^2 - \lambda g_s^{-2/3} \mathrm{Ai} \left(\lambda g_s^{-2/3} \right)^2 \right]$$



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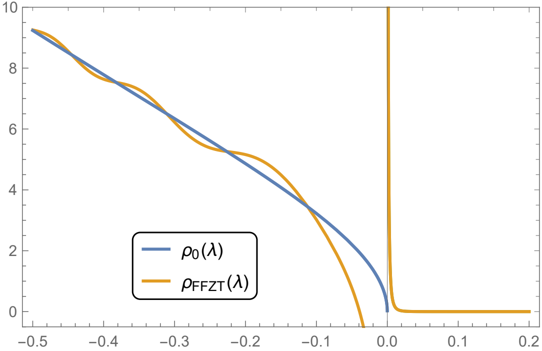
The spectral density

- Non-perturbative contributions add wiggles in the 'allowed' region, and an exponentially decaying contribution in the 'forbidden' region
- The same thing happens when we add FZZT contributions to the JT gravity spectral density



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The spectral form factor

- Recall the definition:

$$\langle Z(\beta+\mathrm{i}t)Z(\beta-\mathrm{i}t)\rangle = \int_{-\infty}^{\infty} \mathrm{d}x_1 \int_{-\infty}^{\infty} \mathrm{d}x_2\, \mathrm{e}^{(\beta+\mathrm{i}t)x_1+(\beta-\mathrm{i}t)x_2} W_2(x_1,x_2) \tag{2}$$

- As it turns out, the spectral form factor is given by a convergent power series because of miraculous cancellations [Blommaert-Kruthoff-Yao]
- The inverse Laplace transform turns the FZZT contribution to the two-point correlator from non-perturbative to perturbative!

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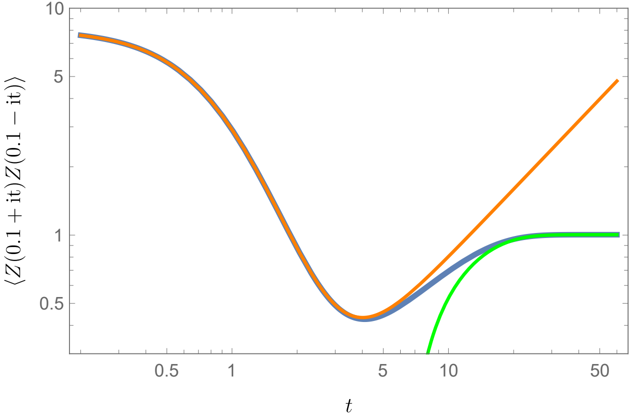
The spectral form factor

- In the Airy case, we see that the FZZT brane contributions (one- and two-instanton) are responsible for the plateau regime of the spectral form factor:



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Summary of the results

- Summary
  1. Using topological recursion, we developed a new systematic technique for computing ZZ non-perturbative data for free energies and correlators in matrix models
  2. We verified the presence of competing FZZT and ZZ effects both in terms of Borel plane singularities and large genus asymptotics
  3. We provided a new (and generalizable) way of computing large genus asymptotics of Weil-Petersson volumes
  4. We were able to reproduce the plateau region of the spectral form factor of JT gravity using FZZT brane contributions
- Outlook
  1. What about multi-instanton sectors? And resonance?
  2. Can we write the full resonant transseries of JT gravity?
  3. How do our techniques extend to spectral curves without an underlying matrix model?

Thank you!

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